Integration of Effort Aspect into Control Strategy Dimensioning and Comparison

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ABSTRACT

The effort required by the actuators of a mechatronic feedback system is considered and integrated into the dimensioning procedure of the controller. Specific measures based on the open-loop frequency response functions of the system are proposed to quantify the performance of a specific control strategy and its corresponding effort requirements. Different control strategies can also be compared.
The proposed measures are applied to disturbance rejection scheme of oscillating plants using feedback control. It is first implemented with a spring-mass-damper system on which two different control strategies are compared. The same procedure is followed with a non-minimum phase system represented by a cantilever beam subjected to bending vibrations and using non-collocated control system.
The obtained results underline the fact that the measures can be used either to guide the dimensioning of the control filter or to quantitatively evaluate and compare different control strategies. It is thus possible to determine which strategy is the most suitable for a specific application in terms of control effort required by the actuating system for a given achievable performance.

1. INTRODUCTION

In mechatronics applications, the energy required by the actuating system depends essentially on the type of sensors and actuators that is used, their location and the employed control strategy. The choice of sensors and actuators implemented is mainly based on practical constraints. The location of the actuating and sensing systems is generally driven by the observability and controllability characteristics of the plant. In loop shaping procedure, the strategy used by the controller, *i.e.* the form of the filter, is usually designed in term of performance and robustness specifications such that settling time, steady-state error, poles location, *etc.*, but rarely in term of effort requirements. It is proposed here to integrate this aspect in the dimensioning step by defining quantitative measures able to guide the determination of adequate control parameters.

Given a specific application, several control strategies can often fulfill the same performance and robustness requirements but with different energy needs. It is thus also interesting to be able to compare different filter forms using measures in order the help the designer to answer to the fundamental question: "Which control strategy is the most suitable for my application?". To achieve this aim, the proposed measures can be employed.

First, the discussed measures are defined. Two examples are then considered in order to illustrate the proposed approach and bring to light the importance of considering the control effort at the stage of the control design.

2. MEASURES DEFINITION

Considering a linear time invariant (LTI) single-input single-output (SISO) stable plant using feedback scheme with LTI and stable controller, one defines two scalar measures: one relative to the controller performance in terms of disturbance rejection, called here P, and another one corresponding to the effort requirement that the actuating system must be able to provide, named E.

The general form of a negative feedback control system, as shown in the figure 1, is considered where G represents the transfer function of the plant seen by the controller, K the transfer function of the controller and G_d the disturbance model. The signal u is the effective command, d the output disturbance and y the delivered output.

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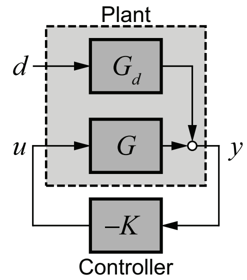

Figure 1: Negative feedback system.

In disturbance rejection applications, the objective of the active system can be seen as an improvement of the characteristics of the plant such that its behavior becomes less influenced by the external disturbances. So the performance of the control strategy can be quantified by the difference between the closed-loop and open-loop configurations of the maximum of the transfer function between the disturbance d and the output of the system y. More specifically, one defines a performance measure P as

$$P := \frac{\|G_{OL}(\omega)\|_\infty}{\|G_{CL}(\omega)\|_\infty} - 1$$

where

$$G_k(\omega) = G_k(s)|_{s=i\omega}$$

for $k = OL, CL$, s corresponding to the Laplace variable and $i = \sqrt{-1}$. Also,

$$\begin{aligned} G_{OL}(s) &= \left(\frac{y}{d}(s)\right)\Big|_{\text{open-loop}} \\ &= G_d(s) \end{aligned}$$

and

$$\begin{aligned} G_{CL}(s) &= \left(\frac{y}{d}(s)\right)\Big|_{\text{closed-loop}} \\ &= S(s)G_d(s) \end{aligned}$$

where S corresponds to the output sensitivity equal to

$$S(s) = (1 + G(s)K(s))^{-1}.$$

The measure used to evaluate the effort provided by the actuating system is defined as the maximum amplitude of the frequency response function between the output of the controller u and the disturbing input d. This effort measure E is formally defined as

$$E := \|K(\omega)S(\omega)G_d(\omega)\|_\infty$$

due to the fact that

$$u(s) = -K(s)S(s)G_d(s)d(s).$$

Both proposed measures are dimensionless and are defined such that they are equal to zero if no control is applied. P increases if the active system induces a positive influence in terms of disturbance rejection and E increases if the amplitude of the controller output becomes larger for a given disturbance input. As they only refer to the frequency response functions of the open-loop system, they thus can be used with experimental data obtained from the open-loop system over a specific bandwidth. Moreover, as both measures depend on the controller parameters, they can be integrated into the optimization phase of the controller dimensioning as well. A practical inconvenient is that generally the transfer function related to the disturbance is *a priori* not known.

3. CASES STUDY

In order to illustrate the approach described above, two cases study are considered. In both examples, the plant consists in a fully observable/controllable underdamped oscillating system. In the first one, a single degree of freedom (SDOF) spring-mass-damper system is used. In the second case, the plant corresponds to a cantilever beam discretized by finite elements method and subjected to bending vibrations.

3.1. Spring-Mass-Damper

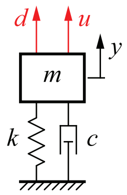

Figure 2: SDOF system.

The figure 2 represents the plant considered. The structural parameters of the plant are its moving mass m, static stiffness k and its viscous damping coefficient c. All these parameters are strictly positive constants. The modal characteristics are the natural frequency ω_n and the damping ratio ζ given by

$$\omega_n = \sqrt{\frac{k}{m}}, \qquad \zeta = \frac{c}{2m\omega_n}.$$

The damping is considered to be sufficiently light so that $\zeta << 1$. The mass is excited by an harmonic disturbing force d. The movements y of the concentrated mass are sensed and filtered by the controller to deliver the reference signal to an actuator acting on the mass with a force u. The dynamics of the sensor, actuator and controller are considered as perfect, *i.e.* without delay. Referring to the representation of the figure 1, in this particular case,

$$G_d(s) = G(s).$$

The objective here is to damp the deviations of the mass from its equilibrium position due to the presence of the disturbance d. Different control strategies can be envisaged. One of the most popular strategies is the direct velocity feedback (DVF) as detailed in [1]. Its popularity comes from its simplicity and efficiency in the case of such collocated system. The feedback plays the role of an additional viscous damper. Direct velocity feedback can be achieved by different ways depending on the signal fed to the controller. Using displacement signal, a lead compensator must be implemented into the controller which actually plays the role of a differentiator. With an acceleration signal, the controller must act as an integrator and if the velocity can directly be measured a simple gain is implemented.
In order to bring to light the importance of the strategy choice on the efficiency but also on the effort, a direct position feedback (DPF) is compared with the previously cited strategy. Direct position feedback acts like an additional spring element over the moving mass.

For simplicity matter, one considers the case for the DVF where the velocity of the moving mass is directly sensed. For the DPF, the position is transmitted to the controller. Thus, in both cases, the control filter consists in a simple gain, so that

$$K(s) = K_c$$

where $K_c \in \mathbb{R}_+^*$.

Direct Velocity Feedback

From the above considerations, the transfer function of the plant is

$$G(s) = \frac{s}{m(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

and the resulting close-loop transfer function between the disturbance and the system output becomes

$$\begin{aligned} G_{CL}(s) &= S(s)G(s) \\ &= \frac{s}{m\left(s^2 + \left(2\zeta\omega_n + \frac{K_c}{m}\right)s + \omega_n^2\right)}. \end{aligned}$$

The maximum of the open-loop and close-loop frequency response functions used to compute the performance measure are located at the natural frequency ω_n and are respectively equal to

$$\|G_{OL}(\omega)\|_\infty = \frac{1}{c}, \qquad \|G_{CL}(\omega)\|_\infty = \frac{1}{(c + K_c)}.$$

The resulting performance measure is equal to

$$P_v = \eta_v$$

with $\kappa_v = \frac{K_c}{c}$.

The control effort measure becomes expressed as

$$E_v = \frac{\kappa_v}{\kappa_v + 1}.$$

It corresponds to the magnitude of the ratio of u over d at the frequency ω_n. It is to note that this measure is upper bounded to 1.
Expressed in function of the performance measure, E_v can be rewritten as

$$E_v(P_v) = \frac{P_v}{P_v + 1}.$$

Direct Position Feedback

In this case, the transfer function of the plant corresponds to the receptance[1]:

$$G(s) = \frac{1}{m(s^2 + 2\zeta\omega_n s + \omega_n^2)}.$$

Under the assumption of light damping, the maximum of the open-loop and closed-loop transfer functions can be approximated by the values

$$\|G_{OL}(\omega)\|_\infty \cong \frac{1}{c}\sqrt{\frac{m}{k}}, \qquad \|G_{CL}(\omega)\|_\infty \cong \frac{1}{c}\sqrt{\frac{m}{k+K_c}}.$$

This gives for the performance measure the following approximation

$$P_p \cong \sqrt{\kappa_p + 1} - 1$$

with $\kappa_p = \frac{K_c}{k}$.

The corresponding effort measure is approximated by

$$E_p \cong \frac{1}{2\zeta}\frac{\kappa_p}{\sqrt{\kappa_p + 1}}.$$

By expressing E_p in function of P_p, one finds that

$$E_p(P_p) \cong \frac{1}{2\zeta} P_p \left(1 + \frac{1}{P_p + 1}\right).$$

This result indicates that the greater the modal damping is, the less control effort is required to achieve the same level of performance.

For both strategies, as the plant is minimum phase[2], there is no limitation over the performance measure. In other words, each strategy is susceptible to achieve the same level of disturbance rejection. However, these results show that this level of performance can not be obtained with the same effort. The ratio of both effort measures expressed in term of performance is equal to

$$\frac{E_p}{E_v}(P) \cong \frac{2+P}{2\zeta}.$$

This implies that

$$\frac{E_p}{E_v}(P) > 1, \ \forall \zeta < 1.$$

Although that both active control strategies could theoretically achieve the same performance, the effort required by the DPF is always greater for every performance level.

This observation shows the importance of considering the effort aspect in the phase of the control design that can have a great influence on the dimensioning of the actuating system.

[1] The receptance is the transfer function defined by the ratio of the displacement over the force.
[2] Do not have any pole and zero located in the right-half plane.

3.2. CANTILEVER BEAM

As second case study, a numerical example is considered. The oscillating system corresponds to a cantilever beam subjected to bending vibrations. At the free end of the beam a disturbing excitation d is applied. This distributed structure is actively controlled using feedback scheme between a movement sensor and a force actuator. The sensor and the actuator are non-collocated. As shown in the figure 3, the actuator is located at a distance $0.6l$ from the clamping point, l being the total length of the beam. The sensor is located at $0.8l$. The dynamics of the sensor, actuator and controller is considered as perfect.

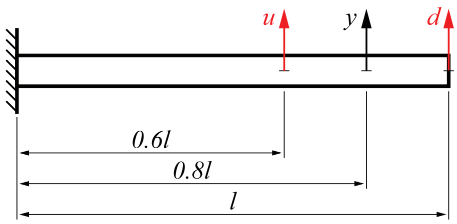

Figure 3: Cantilever beam model.

The beam has a length equal to 100 mm and a circular cross section of 10 mm diameter. Its Young's modulus, Poisson's ratio and density are, respectively, equal to 210 MPa, 0.3 and 7850 kg/m^3. The structural damping is modeled as proportional to the mass and stiffness matrices, M and K, such that the viscous damping matrix C being equal to

$$C = 0.3 \cdot 10^3 \cdot M + 0.5 \cdot 10^{-6} \cdot K.$$

The plant is discretized using FE-beam elements and only its first five bending modes are considered. The modal parameters of these first modes are listed in the table 1.

Table 1: Modal parameters of the first five bending modes of the plant.

Mode	**Natural frequency (Hz)**	**Damping ratio (%)**
1	720	3.4
2	4367	1.2
3	11674	2.0
4	21576	3.5
5	33466	5.3

In this example again, the objective is to damp the deviations from the equilibrium position of the beam subjected to disturbances. For this purpose, the strategies used previously are employed. The main attractivity of these strategies is due to the fact that they do not require any model of the plant in comparison with model-based strategies such as LQR, H_∞, *etc.*. However, the plant considered here is non-minimum phase and applying the proposed strategies on it could give rise to severe negative effects. This is why before going further some considerations are required.

Plant Analysis

Due to the non-collocation configuration between the sensor and the actuator, pole-zero flipping phenomena[3] can occur between the frequency range considered leading to non-minimum phase open-loop system and restricting the controller bandwidth and its robustness.
Physically, the pole-zero flipping is related to the sensor/actuator arrangement and the modal shape of the natural modes of the structure. In this example, such phenonemon happens if two subsequent mode shapes present an additional nodal point located in between the actuator and the sensor leading to a 180 deg phase lag of the actuator signal with the sensor signal. If this phenomenon occurs, the related mode is excited by the active system instead of being damped and can lead to instability.
In the proposed example, pole-zero flipping happens between the first and the second modes. This is due to the nodal

[3]See [1] for more details.

point of the second bending mode shape located between the actuator and the sensor. Thus, instead of being damped, the second mode is excited by the controller and decreases the stability margins. This produces a negative effect on the disturbance rejection performance as the deviations resulting from the excitation of the second mode are amplified.

Direct Velocity Feedback

Like for the spring-mass-damper system, a velocity feedback is considered so that the controller consists in a simple gain,

$$K(s) = K_c.$$

As previously explained, due to the non-collocation between sensor and actuator, the open-loop system is non-minimum phase and so the gain margin is finite. The Nyquist stability criterion is used to determine the range over which the controller gain can be varied without generating instability. For the proposed example, the stability limit is reached for a gain equal to 262 Ns/m. By varying the gain K_c up to this value, one obtains for the measures the curves shown in the figure 4.

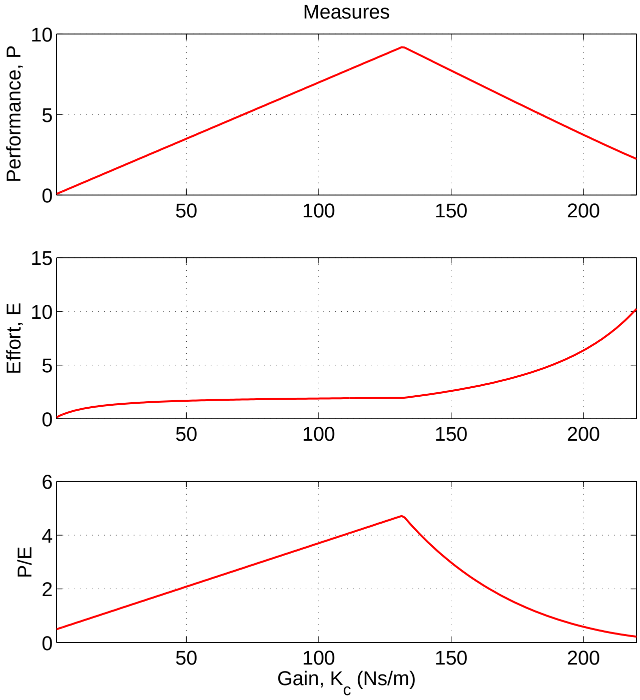

Figure 4: Measures obtained with the direct velocity feedback.

The performance reaches the maximum value of 9.1 for a gain equal to 132 Ns/m. Beyond this point, the excitation of the second mode generates amplitudes larger than the first mode and the performance measure starts decreasing.
The effort measure shows a smooth increase up to a gain value around 134 Ns/m. After this value, the effort required to damp the first mode becomes less than the effort necessary to excite the second one and the effort starts to increase almost exponentially.
The ratio of both measures allows to point out the gain providing the best performance-to-effort ratio. In this case, this optimal gain coincides with the greatest performance and the maximum of the performance-to-effort ratio is equal to 4.7.

Direct Position Feedback

Like in the first example, the active system is considered sensing the displacements so that the controller consists in a simple static gain.

By applying the same procedure as for the DVF, the maximum control gain is found to be equal to $18.3 \cdot 10^6$ N/m. The obtained values of the measures are shown in the figure 5. The maximum of the performance occurs for a value of K_c equal to $12.5 \cdot 10^6$ N/m and is equal to 2.3. If the performance measure stays in the same order than for the DVF, the required effort is much higher. As a consequence, the performance-to-effort ratio is much smaller than for the DVF. This ratio presents an optimum equal to $22.0 \cdot 10^{-3}$ corresponding to a gain of $6.7 \cdot 10^6$ N/m. In comparison with the DVF, this optimal value is more than 200 time smaller.

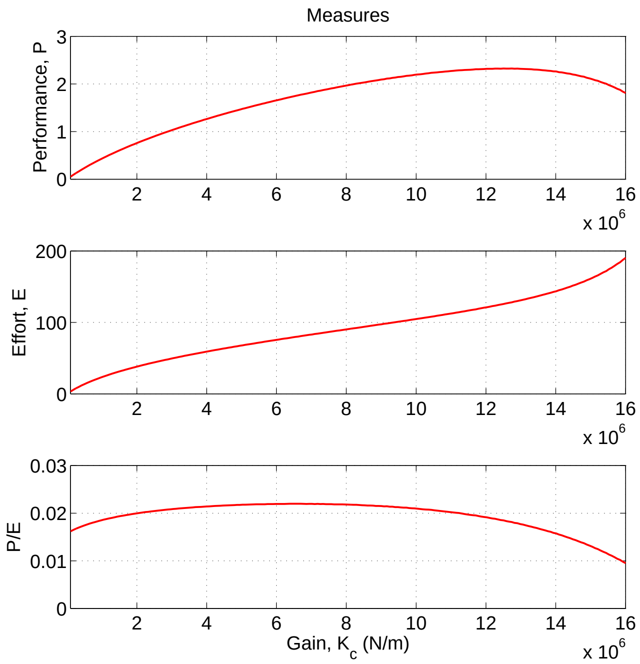

Figure 5: Measures obtained with the direct position feedback.

This observation confirms the results obtained with the first case study. So one can state that, when used for disturbance rejection purpose, the DPF control requires much more effort than the active damping strategy and would lead to an overdimensioning of the actuating system in order to achieve the same performances.

4. CONCLUSION

In the present article, two measures are defined in order to quantitatively evaluate and compare different active control strategies dedicated to the disturbance rejection scheme. One measure quantifies the positive influence of the control over the behavior of the resulting system. The other one gives a measurement of the effort required by the actuating system to apply the control strategy on the plant.

Two examples have shown that through these measures the ability of a given strategy to improve the dynamics of a plant against external disturbances can be quantified. Also the effort required by the actuating system can be evaluated. So, for a specific application, such measures can be employed in the control design step using simulated or experimental open-loop data in order to compare different potential control strategies. They can also be integrated in optimization scheme for the control dimensioning procedure.

References

[1] *A. Preumont: "Vibration Control of Active Structures, An Introduction"* 2nd *edition, Kluwer Academic Publishers, Solid Mechanics and its Applications, Vol. 96, 2002.*