XXI IMEKO World Congress “Measurement in Research and Industry”
August 30 − September 4, 2015, Prague, Czech Republic

THE SIGNAL RELATION DIAGRAM AS A METROLOGICAL TOOL – ELEMENTS AND SYNTHESIS

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Abstract – Relations and correlations between signals are important issues in modelling and simulation practice, especially in the field of Metrology. As a formal and descriptive "language", the so-called signal relation diagram (SRD) represents relations graphically. It visualises relational structures and generally makes relations more transparent. Signal relation diagrams base on logical and mathematical phrasing. Just a few basic elements are necessary. A straightforward synthesis unites these elements in typical arrangements in a holistic and consistent way. They serve as basic elements and structures in varied fields of Metrology, for modelling, description, simulation, analysis, reconstruction, filtering, verification and so on. The key task is information, understanding and knowledge.

Keywords: quantity model and process model, signal relation and correlation, system properties and behaviour, system structures and interconnections

1. INTRODUCTION

There are several graphic-based tools, which support the description and understanding of scientific and technological phenomena. One of them is the *Signal Relation Diagram* (SRD). Its task is twofold.

Firstly, it is a medium, which methodically describes *nets of interrelations* of continuous and / or discrete time signals according to the *principle of cause and effect*. The standard question therefore is always: How does a signal influence another signal, how is the second signal related to the first one? Those relations are represented graphically in the signal relation diagram, which is easily interpretable and therefore a most welcome base of thorough discussions in all fields of Science and Technology.

Secondly, it is a catalyst for an immediate creation of computer instructions, which allow simulation and analysis of quantities and processes on the one hand and model based activities by *observers* and *controllers* on the other hand.

This dual aspect is crucial, since comprehension and implementation are closely linked. In this respect, the signal relation diagram is one of several formal "languages", appreciated by countless fields of application.

More to the point: We recognise signals as models of quantities in and around a *real-world process*, revealing its *properties* and *behaviour*. Therefore, a signal relation diagram is a qualitative or quantitative *graphical model* or *map*. Establishing such a model means that we systematically define and describe signal relations in and around the process by analytical and / or empirical means. Such a model is well defined, although it always remains more or less rudimental and incomplete, by necessity or on purpose. The model is injective; it represents a one-to-one correspondence. Therefore, the logical and mathematical language of the model with *its* terms and definitions is transformable into one of the graphical languages with *their* terms and definitions and vice versa (Cause and Effect Principle, Conservation Concept, Mason Gain Rule; Block Algebra). The usual mathematical constraints of model development apply throughout. On the other hand, seemingly intransparent mathematical structures gain clarity by graphical representations. Even certain analogies and symmetries may emerge, which enable additional structural insight.

A signal relation diagram is not a tool of its own. There is no special theory. Only a few guidelines of implementation have to be observed. But, here too, habits from the past dominate the scene, they are not always rational.

In summary, the model, on which such a diagram is based, describes *relations* and *correlations* between *signals* only: *neither more, nor less*! Consequently, signal relation diagrams have nothing in common with those block diagrams, which represent real-world items (object, entity, artefact, apparatus, instrument, person, particle, device, article, product, processor, subject, individual, target, body, matter, asset, constituent, element, field, organisation, reality, cosmos), although this keeps being suggested. Of course, stated relations and correlations are always *ascribed* to certain processes or sub-processes.

It has to be mentioned that quantities like mass, energy, power, momentum, impulse and especially their flow in a dynamic process, are considered as ordinary quantities too. They appear in the context of so-called back-loading effects, represented by *backward directing signal paths*, "back" here in a figurative sense.

Furthermore, signals may not just be models of *real-world quantities* (hard quantities), but also models of *abstract definition quantities* (soft quantities). Examples: efficiency value, performance index, observability, comfort factor, signal to noise ratio, effort, error, residuum, uncertainty, variance, reliability, probability density function and so on. They are well-defined, and thus visualisable in signal relation diagrams. They are *observed* and *measured* by *indirect* means.

Sometimes it may help to consider signals as *information* and to interpret signal relation as *information relation* or even as *information fusion*.

The signal relation diagram is also known under several other terms (synonyms): relationship diagram, signal graph, signal coupling diagram, signal flow diagram, flow chart, context diagram, signal path diagram, signal diagram, integration diagram, block diagram, data graph and so on.

But, since models just describe interrelations between signals, the term *signal relation diagram* (SRD) seems the most appropriate one. It is not very common and there is no established terminological standard.

On the other hand, the frequently used term signal "flow" diagram is misleading indeed: *Nothing flows*. This in contrary to a *wiring diagram* in Electronics, to a *piping diagram* in Hydraulics, to a *free body diagram* in Mechanics, to a *binding diagram* in Molecular Modelling, to a *workflow diagram* in Business Process Modelling (BPM), and so on. Even if a signal models a flow, the *influence* or *impact* of the quantity flow on the quantity fluid level or gas pressure is meant, and not the flow itself.

We know useful model types with other graphical representations concerning signal relations. There is for example the *Bond Graph* [1]: It concentrates on the omnipresent, intrinsic *conjugate pair* of physical quantities, which represents the generalised *effort quantity* and the generalised *flow quantity*. This accounts for the important, but often neglected role of mass, energy and impulse as carrier quantities of information quantities, also neglected frequently in Metrology.

Another representation is the popular Cause and Effect Diagram (CED) (fishbone diagram, Ishikawa diagram). It merely offers limited qualitative, and no quantitative possibilities [2].

There are more, highly specialised signal relation diagrams, like the Mohr Circle, where stress quantities induced by force quantities in material are handled and visualised graphically in the important three-dimensional case.

Prerequisite of any type of signal relation diagram is a systematic and consistent model, which is developed on the basis of logical and mathematical structures and parameters. Even if values of the parameters are still unknown, intense discussion about a process is possible already. Structures of relations are prime issues, when we start modelling. This makes signal relation diagrams valuable in fields, where numerical values of quantities and parameters are not or not yet available, as in Metrology or Humanities for example. Of course, eventually we need in-depth process knowledge for quantitative information. Consequently, signal relation diagrams are known to grow according to the gradual development of demand and insight.

Although there are only four basic elements concerning the signal relation diagram and though the systematic rules of relating signals are rather simple, design, handling and analysis are often demanding. They should be practised, so we can gain operating experience.

2. SIGNALS AND SYSTEMS – MODELS OF QUANTITIES AND PROCESSES

At the beginning of each *model development*, we must acknowledge and postulate the difference between the *process to be modelled* and the *system as model of the process*. Process and system are items in two different domains. There is the *real-world process domain* and the *abstract system domain*. The mentally designed system describes the process, but of course, is not identical with it. Properties and behaviour, which are identified by the model analytically and / or empirically, are ascribed to the process of interest, but, of course, do not have any influence on the process whatsoever. So, the description and identification of the process is a purely mental procedure, its informational content is imbedded in the system.

The same is true for the models of quantities. There are the *real-world quantities* to be modelled. We call these models *signals* in the *abstract signal domain*. We ascribe the identified properties of the signals to the real-world quantities.

3. THE FOUR ELEMENTS OF THE SIGNAL RELATION DIAGRAM

A signal relation diagram uses only four graphical tools, visualised by four metaphoric icons (symbols).

- *relation line*, identifying signals
- *connection circle*, connecting signals
- *branch point*, branching signals
- *relation block*, relating signals

We have to mention one major restriction concerning these four elements: They are only partly suitable for systems with *distributed parameters*, described by *partial differential equations*, since they can be drawn only in the single dimension of the paper or screen surface. Unfortunately, other popular tools, for example those with colour assisted three-dimensional capabilities (Finite Element Method (FEM), Mohr Circle), are only partly able to fill the breach.

Yet the four graphic elements suffice for linear and nonlinear, multivariate dynamic systems. They combine ubiquitously and are thus able to visualise models in a broad field of applications. In particular, they foster the modelling of multivariate situations with signal sets, signal vectors and signal ensembles. The relations and correlations between these signals may be dynamic and nondynamic, continuous time and discrete time, linear and nonlinear, deterministic and probabilistic, to mention only the most important ones.

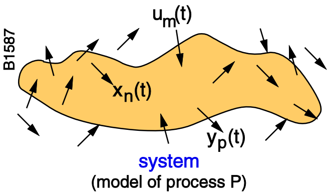

Figure 1. Symbolic visualisation of signals (models of quantities) in and around a system (model of a process)

This simple graph (Figure 1) already indicates, which signals, represented by the lines with arrows, could be important for an intended model (set of relations), and which are not. We recognise signals, surrounding the system, which do not take any influence at all, signals, which do have influence indeed and act on the system, signals, which represent the state in the interior of the system, and signals, which influence the exterior world.

This sketch has to be simplified and standardised (Figure 2). The exterior signals are deleted, even though the knowledge about their existence might still be important for future documental purposes.

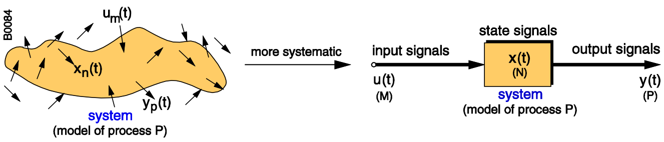

Figure 2. Output signals **y**(t) depend on input signals **u**(t)

This is the first step in the direction of a systematic and coherent visualisation and the definition of influencing and influenced, independent and dependent, input and output signals. They are each combined in an input vector **u**(t) and an output vector **y**(t). The inner signals of the system are regarded as state signals in the state vector **x**(t).

4. PROPERTIES OF THE FOUR ELEMENTS OF THE SIGNAL RELATION DIAGRAM

There are many possibilities to apply and interpret these four elements. But particular properties prevail, because they are not task and application oriented. They represent basic aspects of Signal and System Theory.

4.1. Relation Line

Relation lines (Figure 3) for signals with directing arrow heads. They symbolise abstract models (signals) of real-world quantities (physical, chemical, biological, medical, economical, sociological, psychological, educational, cultural, etc. quantities).

$$x(t) \qquad \mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \\ \vdots \\ x_N(t) \end{bmatrix} = \begin{bmatrix} x_1(t) & x_2(t) & \ldots & x_n(t) & \ldots & x_N(t) \end{bmatrix}^T$$

Figure 3. Scalar and vectorial relation lines

We describe a *single* (1) *signal* (scalar signal x(t)) by a *thin type line* and *multiple* (N) *signals* (vector signal **x**(t)) by a *bold type line*. The vector signal **x**(t) contains (N) signals $x_n(t)$ of any type as its elements. This distinction is not done systematically in literature, though it fosters clarity.

Often, we have to visualise and to denote relations between probabilistic signals (probabilistic events). Any single probabilistic signal $x_h(t)$ has always to be considered as a sample (element) in a signal ensemble (group, family, collective) (Figure 4).This ensemble deserves its own notation too: {x(t)}.

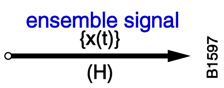

Figure 4. Ensemble relation line for probabilistic signals

It contains (H) probabilistic signals $x_h(t)$ as its elements. They are always of the same type, but each changes randomly as an individual (random walk). So, statistical operations across such a signal ensemble {x(t)} are feasible (Figure 5).

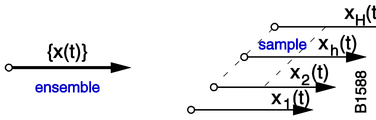

Figure 5. Ensemble and sample lines for probabilistic signals

As a consequence we have to use ensemble vectors {**x**(t)} with multivariate systems. They contain (O) signal ensembles $\{x_o(t)\}$ as their elements.

Annotation of Signals

Signals are labelled by noncommitted symbols delivered by Signal and System Theory or by common symbols from applicational fields.

Further information may be added by the total number of the signals (elements) within the signal vector.

Sometimes physical units of the quantities are applied.

If the model of a process is designed for, or transformed into another domain, annotations of the signals indicate this fact by corresponding symbols and units.

The appearance of time-continuous and / or time-discrete signals can be indicated too. The annotation differs only slightly: Replace the symbol of a time-continuous signal x(t) by $x(k{\cdot}\Delta t_s)$ or in short by x(k) for time-discrete signals, where Δt_s is the sampling period.

4.2. Signal Connection Circle

Connection circles relate signals directly, outside the signal relation block, by the four basic mathematical operations: *linear connection* (addition, subtraction) or *nonlinear connection* (multiplication, division). The connection circle contains the operation symbol. Signals to be connected point to the circle; resulting signals lead away from the circle.

Linear Connection

Circle with capital sigma (Σ) for the summation operation (Figure 6).

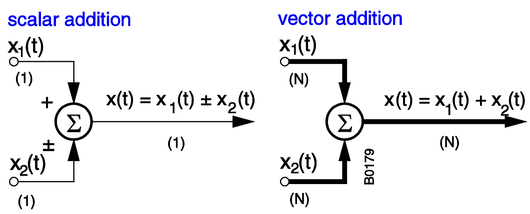

Figure 6. Linear signal connection

Nonlinear Connection

Circle with capital pi (Π) for the multiplication operation and ÷ for the division operation (Figure 7).

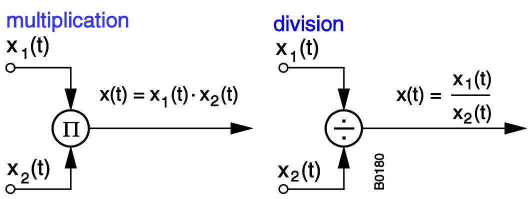

Figure 7. Nonlinear signal connection

Exception: The multiplication / division of a signal (variable) with / by a constant scalar is not a connection and thus is symbolised by a parameter operation in a signal relation block.

There is a special type of connection (composition) or of disconnection (decomposition) of signals respectively: Firstly, we *merge* two or more scalar signals or vector signals into one single vector signal of corresponding size. Secondly, we *demerge* one signal vector into two or more scalar signals or vector signals of corresponding size (Figure 8). These procedures are symbolised by vertical *merge bars* and *demerge bars* respectively.

Note that these connecting and disconnecting procedures foster only a clearer arrangement of multiple signals. They have nothing to do with the signals themselves.

merge

$$\begin{bmatrix} \mathbf{x}_1(t) \\ \mathbf{x}_2(t) \end{bmatrix} = \begin{bmatrix} x_{11}(t) \\ x_{12}(t) \\ x_{21}(t) \\ x_{22}(t) \\ x_{23}(t) \end{bmatrix} = \mathbf{x}(t)$$

demerge

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{x}_1(t) \\ \mathbf{x}_2(t) \end{bmatrix} = \begin{bmatrix} x_{11}(t) \\ x_{12}(t) \\ x_{21}(t) \\ x_{22}(t) \\ x_{23}(t) \end{bmatrix}$$

Figure 8. Merging and demerging of signals and signal vectors

4.3. Signal Branch Point

A *branch point* branches the *effect* of a signal into several directions without changing its qualitative and quantitative character at all, obviously unlike a branching of *flows* of matter and energy in a "wiring" diagram.

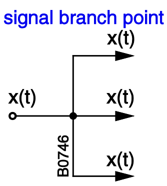

Figure 9. Branching of signals and signal vectors

Caution: Two crossing signal lines without the branch point are not related at all (Figure 10).

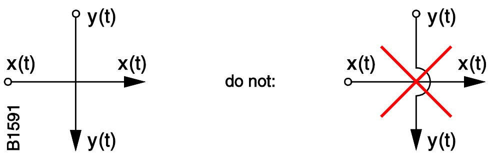

Figure 10. Crossing of signals and signal vectors
in a signal relation diagram

4.4. Signal Relation Block

Relations: Arbitrarily many signals, models of quantities, may be related to each other: one-to-one, one-to-many, many-to-one and many-to-many relationships. We describe each relation by a set of mathematical formalisms, call such a set *model of a process* (= *system*), and visualise it graphically by a *square* or *rectangular signal relation block*. There may be (M) input signals **u**(t), effective preferably from the left, (N) state signals **x**(t) within the block and (P) output signals **y**(t), effective preferably to the right (Figure 11).

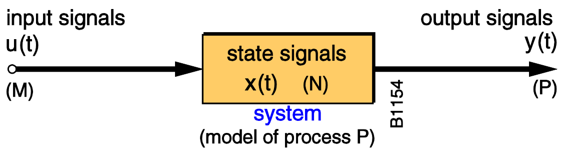

Figure 11. Signal relation block
as representative of a system

The arrowhead at the end of a signal relation line marks the direction of relations. There are *independent signals*, when we look at the inputs of relation blocks, connection circles and branch points, on which they have an "impact". And there are *dependent* signals, when we look at the outputs of relation blocks, connection circles and branch points, by which they are "affected". Therefore, in a usual network of several relation blocks, a signal is independent *and* dependent at the same time, just depending on the point of view.

Note: Never ever are there bidirectional signal relation lines. Whenever *loop*, *feedback* or loading relations arise somewhere, which is often the case, they get signal lines of their own with arrows pointing backward (generally to the left) (Figure 12).

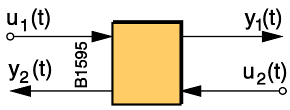

Figure 12. Exclusively unidirectional signal lines
despite of back-directing relations

In principle all mathematical operations in a block are permitted. Again, we speak of relation blocks, and not of process blocks. *Logical* and *mathematical procedures* are meant and not *real-world procedures*.

The simplest case is the relation (transfer, transformation, mapping, dependence) between one output signal and one input signal (*single input, single output*; SISO) (Figure 13).

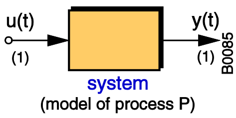

Figure 13. Simple relation between
one input signal and one output signal.

Starting the design of a model, a simple relation block with input and output signals just indicates that there may be relations between these signals; neither more nor less. This means that we may already design and discuss large signal relation diagrams, even though we, at this date, lack quantitative information concerning structure and parameters of the supposed model. This situation can be found in most fields especially beyond Sciences and Technologies.

In order to roughly indicate the level of knowledge about assumed relations, it is customary to distinguish between black, grey and white relation blocks (Figure 14).

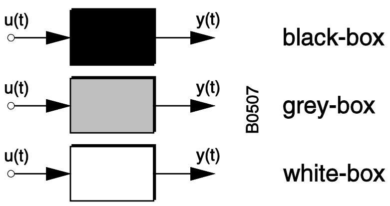

Figure 14. Black, grey and white relation blocks

When we solve a set of mathematical descriptions within a signal relation block with respect to the output signals, we get to know, how systems react to selected input signals **u**(t) by output signals **y**(t) under certain circumstances. We call such a obtained *solution* the *behaviour* of this system. This may be one of the following *standard behaviour* like the *impulse response behaviour*, the *step response behaviour*, the sine-wave response behaviour, the *probabilistic response behaviour*, and so on. Or it will be a dedicated solution serving special needs.

For dynamic systems the solution of an ordinary differential equation is notified by integrator blocks (solution blocks) (Figure 15). Their number match the degree (N) of the differential equation. The initial values **x**(0) at time instant t_0 of the (N) state signals **x**(t) are indicated at the top of the block.

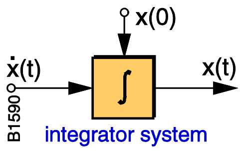

Figure 15. Integrator block as the representative of the solution procedure for an ordinary differential equation

5. DEALING WITH SIGNAL RELATION BLOCKS

5.1. Composition of Signal Relation Blocks

We may *compose* (merge, condense) selected relation blocks with connection circles and branch points in order to hide them in one larger, single block. This is just an organisational means aimed at getting a better overview and a convenient analysis (Mason Gain Rule; Block Algebra, System Theory).

We reach such arrangements (nets, fusions), always based on the principle of cause and effect, by means of three and only three connection types. They are shown here exemplarily for two systems (Figure 16):

- series connection
- parallel connection
- feedback (loop) connection

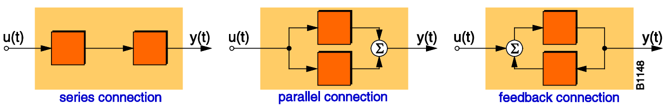

Figure 16. Three connection types of two systems

We often want to arrive at special *canonical structures* on the level of Signal and System Theory with the help of such compositions. Most useful are the structures of the State Space Description (SSD) as models of the linear time invariant (LTI) dynamic processes, for example in the time domain (Figure 17). The basic set of equations is given in vector-matrix-form by:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}\,\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$$

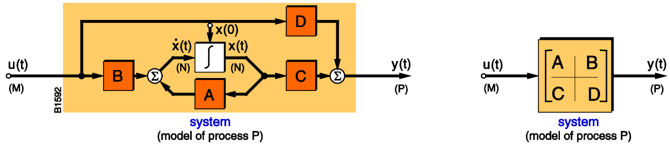

Figure 17: Canonical signal relation diagram of the linear, time invariant (LTI) dynamic system

It is remarkable that such a concentrated structure of a highly complex process model, uses only the four elements of the signal relation diagram mentioned above. It is also remarkable that this structure is applicable nearly in any field of scientific endeavour.

There are more canonical forms of systems for special needs. They are presented by Signal and System Theory [3; 4; 5].

5.2. Decomposition of Signal Relation Blocks

It is feasible to *decompose* (partition, factorise, demerge, fragment, split) a relation block together with its set of mathematical equations into two or more sub-blocks with respect to a finer level of details and / or to predefined properties of interest. Again, the decomposed blocks will be arranged in series, parallel or feedback connection.

For example, the model of a nonideal process P is decomposed: The main, nominal block PN incorporates the postulated ideal, nominal properties (behaviour) of the process and the separate error block E unifies the *nonideal* properties of the process, which provokes the overall, nonideal behaviour (Figure 18).

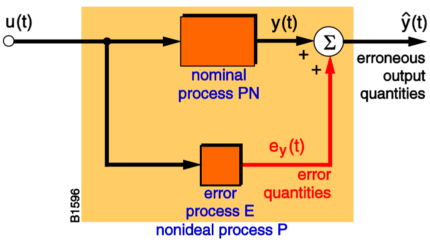

Figure 18. Decomposition of nonideal process

Thereby, the analysis of errors and uncertainties becomes much more persuasive. Such a perception is an important issue in Metrology.

5.3. Inversion of Decomposed Systems

The Cause-and-Effect Principle dominates Natural Science and Technology. Normally, we know the effects of a running process but not its causes. Often we would like to infer from an effect to its cause. Such an inference is possible, as soon as we know the model. Sometimes, especially in Metrology, we call this inference "reconstruction". Mathematically this problem is solved by the inversion of the model, a procedure, which is not always possible.

Now, what does this look like for more complex systems, which are composed of sub-systems? The inverted structures depend strongly on the cause-and-effect structures as we may expect.

If the response (effect) y(t) of a system S is known and we want to infer on to the original excitation (cause) u(t), a reconstruction process R solves the problem by inversion of the model of the system.

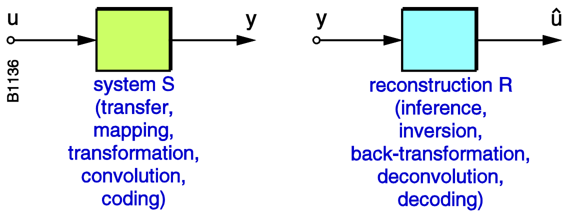

Figure 19. System and its reconstruction

Now, if two sub-systems are connected, then how does this inversion procedure look like? At first: We have the three connection types, series connection, parallel connection and feedback connection (Figure 16). Question: Are there correspondences between the connection types of the inverted system and the original system? Yes indeed, this is the case. We get the following results, where g is a transfer value or a transfer function, representing the relation between the respective input signals and output signals (Figure 20):

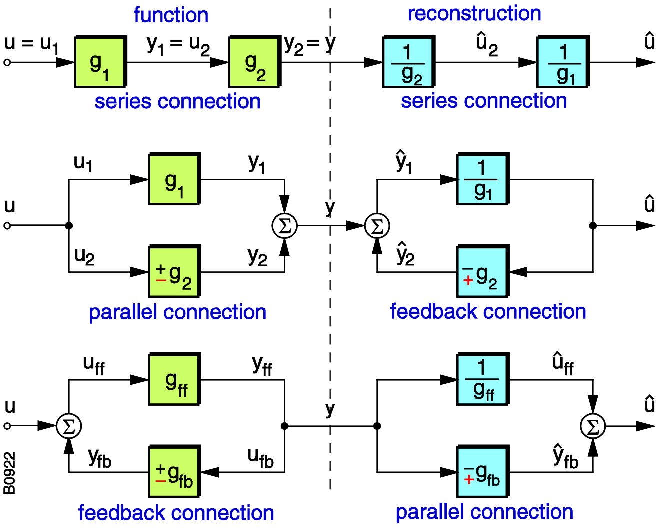

Figure 20. Reconstructed structures of the three connection types, series, parallel and feedback connection

Notice the obvious vertical symmetry line in the signal relation diagram! The following correspondences are visible:

- The inverse of a series connection is a series connection.
- The inverse of a parallel connection is a feedback connection.
- The inverse of a feedback connection is a parallel connection.

We may derive this intuitively too, by virtually going back on the effecting paths from the output quantity y (retrospection, review, inference) and by reasoning, how we might get from the given output signal y to the sought-after input signal $\hat{u}$.

Moving *against* the original signal direction, the transfer response value g of a block has to be inverted. Moving *with* the original signal direction, the transfer response value g of a block has not to be inverted. Branch points and connecting points remain unchanged. One of the two signs is changed at a connecting point.

Even the inverse of more complex linear systems must have systematic and symmetric structures too. The example of a sensor process S with two input quantities, two output quantities and cross disturbances (cross talk) demonstrates this fact (Figure 21). The reconstruction process R must always be the inverse of the sensor process S.

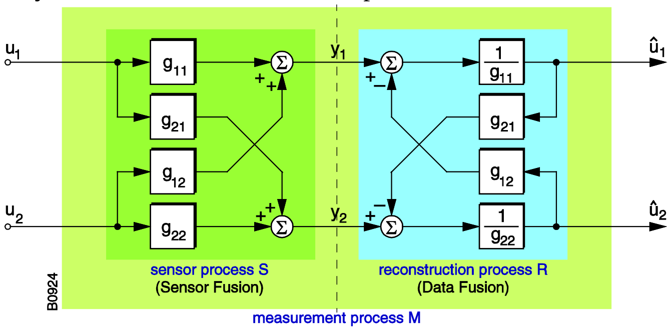

Figure 21. Inversion of the model of a sensor process with two input signals.

6. CONCLUSION

A signal relation diagram (SRD) is an important tool to visualise complex processes. Main elements are signals and systems. Relations between these signals represent the systems. This is quite a new and straight forward perspective.

It is shown that ubiquitous connections and disconnections of systems create interesting new structures, which get dedicated meanings in different fields.

Basis of all these definitions and procedures are always Signal and System Theory.

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